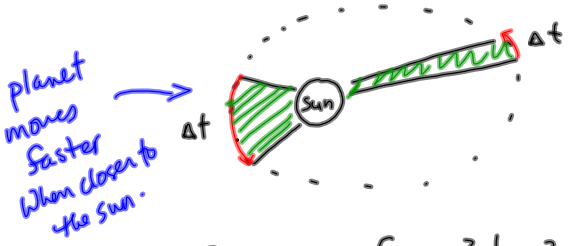
§12-1 Newton's Law of Universal Gravitation

Kepler's Laws

1. Planets move in elliptical orbits.
2. Planets sweep equal areas in equal times:



3. The ratio of r^3/T^2 is constant (for a given central body)

$$K_{sun} = 3.35 \times 10^{18} \frac{m^3}{s^2}$$

$$\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$$

Newton

F_g is directly proportional to m_1 and m_2

F_g is inversely proportional to the square of the separation

$$F_g \propto \frac{m_1 m_2}{r^2}$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

Where m_1 and m_2 are masses (kg)

r is the separation (centre to centre) (m)

F_g is the force of gravity (N)

$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

See p955

Newton's Hypothesis:

Newton said that $F_g = F_c$

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2} = m4\pi^2 r f^2$$

(m: orbiting mass)

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Find the mass of sun using the Earth's radius of orbit and its period of orbit:

$T = 365.25\,d$

$r = 1.49 \times 10^{11}\,m$

$m_{sun} = ??$

$365.25\,d\left(\frac{24\,h}{1\,d}\right)\left(\frac{3600\,s}{1\,h}\right)$

$= 31557600\,s$

Using Newton's Hypothesis:

$F_g = F_c$

$$\frac{G m_{sun} m_{earth}}{r^2} = \frac{m_{earth} 4\pi^2 r}{T^2}$$

$$m_{sun} = \frac{4\pi^2 r^3}{G T^2}$$ ← Kepler's constant for the sun.

$$m_{sun} = \frac{4\pi^2 (1.49 \times 10^{11}\,m)^3}{\left(6.67 \times 10^{-11}\,\frac{N \cdot m^2}{kg^2}\right)(31557600\,s)^2}$$

$m_{sun} = 1.97 \times 10^{30}\,kg$

TODO:

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② Planetary Motion Handout/Animation